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# Effects of Planetary Gear Ratio on Mean Service Life

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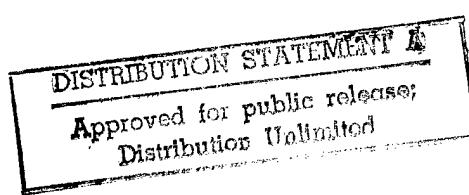
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## EFFECTS OF PLANETARY GEAR RATIO ON MEAN SERVICE LIFE

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### ABSTRACT

Planetary gear transmissions are compact, high-power speed reductions which use parallel load paths. The range of possible reduction ratios is bounded from below and above by limits on the relative size of the planet gears. For a single plane transmission, the planet gear has no size at a ratio of two. As the ratio increases, so does the size of the planets relative to the sizes of the sun and ring. Which ratio is best for a planetary reduction can be resolved by studying a series of optimal designs. In this series, each design is obtained by maximizing the service life for a planetary with a fixed size, gear ratio, input speed, power and materials. The planetary gear reduction service life is modeled as a function of the two-parameter Weibull distributed service lives of the bearings and gears in the reduction. Planet bearing life strongly influences the optimal reduction lives which point to an optimal planetary reduction ratio in the neighborhood of four to five.

### NOMENCLATURE

#### Variables

a	-	gear addendum (mm,in) and bearing life adjustment factor
C	-	dynamic capacity (kN,lbs)
E	-	elastic modulus (MPa,psi)
f	-	face width (mm,in)
F	-	load (kN,lbs)
K <sub>f</sub>	-	stress concentration factor
l	-	life (10 <sup>6</sup> cycles and hours)
n	-	gear ratio relative to the arm, number of planets
n <sub>a</sub>	-	actual transmission gear ratio
N	-	number of gear teeth

P <sub>d</sub>	-	diametral pitch (1.0/inch)
R	-	gear radius (mm,in) and reliability
Y	-	Lewis Form Factor
φ	-	pressure angle (degrees, radians)
v	-	Poisson's Ratio
ρ	-	radius of curvature (mm,in)
σ	-	bending stress (Pa, psi)
σ <sub>H</sub>	-	Hertzian contact stress (Pa, psi)
ω	-	angular velocity, speed (rpm)
ω <sub>b</sub>	-	bearing load cycle speed (rpm)
Σ	-	central angle between two adjacent planet center lines with the input shaft center (radians)

#### Subscripts

av	-	mean
d	-	dynamic
o	-	output
pl	-	planet
r	-	ring gear
s	-	sun gear
1	-	pinion
2	-	gear
10	-	90 percent reliability

#### Superscripts

b	-	Weibull slope exponent
p	-	load-life exponent

## INTRODUCTION

Planetary gear transmissions offer the user a moderate gear reduction with a high power density (Fox, 1991) (Lynwander, 1983). By carrying multiple planet gears on a rotating arm, load sharing is enabled among the planets (Fox, 1991) (Lynwander, 1983). The symmetric placement of the planets about the input sun gear provides radial load cancellation on the bearings which support the input sun and the output arm (Fox, 1991) (Lynwander, 1983). The fixed internal ring gear support also has no net radial load. With near-equal load sharing in medium to fine pitch gearing, a compact reduction results. Planetary reductions are often found in transportation power transmissions due to this weight and volumetric efficiency (Fox, 1991) (Rashidi & Krantz, 1992).

Much of the published design literature for planetary gearing focuses on the kinematic proportioning of the unit to achieve one or more reductions through the auspicious use of clutches and brakes (Muller, 1982) (Tsai, 1987).

Recent literature on planetary gears has focused on the dynamic loads in the transmission with measurements of load sharing and the variations of load in specific units (Rashidi & Krantz, 1992) (Donley & Steyer, 1992) (Saada & Velez, 1995) (Kahraman, 1994). Monitoring of the dynamic loads in a planetary has also been proposed as one method of determining the need for preventive maintenance in the transmission (Chin, et al, 1995).

While the reduction of dynamic loads in a planetary transmission is an important task, these studies do not indicate which ratio is best suited for a planetary transmission. Studies of rotating power in the planetary have indicated that as the ratio is increased, the percent of rotating power in the unit reduces (Lynwander, 1983). This suggests that the best ratio for a planetary reduction is the highest possible, which is reached with the largest planet gears. Addendum interference between the planets determines this limit. However, when one considers the size of a planetary reduction required to transmit a given power level at some input speed, the loading on the gears and bearings in the reduction become an important factor as do the component lives under load (Savage, et al, 1989 & 1994a).

Since aircraft and automotive transmissions can see service in excess of their nominal design lives, periodic maintenance is provided throughout their lives (Chin, et al, 1995) (Savage & Lewicki, 1991). The service life of a transmission between maintenances is a design variable which one would like to maximize for a given size and power.

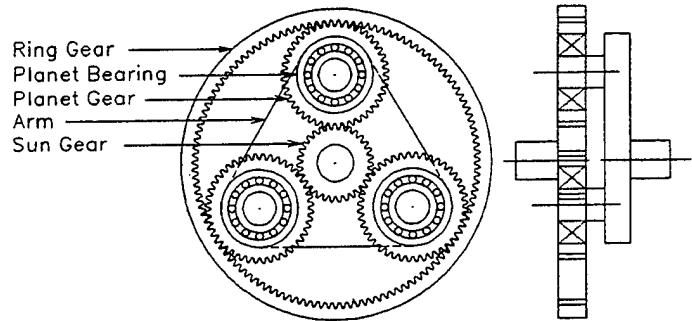
Programs have been written to optimize transmissions for service life (Savage, et al, 1992, 1993, 1994b) (Rubadeux, 1995). The service life of the transmission is modeled as a function of the service lives of the components which have a two-parameter Weibull distribution. The critical components for this calculation are the bearings and the gears in the transmission. A mean life of the transmission is determined from the mean lives of the critical components under load.

In this paper, the influence of speed reduction magnitude on the service life of a planetary gear reduction is investigated for reductions with similar components. An optimal gear reduction for a planetary gear set is sought considering the size and capacities of the components. For a fixed power level and transmission size, the life is charted versus reduction ratio for a fixed input speed and three, four and five planets.

## PLANETARY CONSTRAINTS

In comparing the lives of similar transmissions, one needs to specify the conditions of similarity. The planetary gear reductions considered in

this work are single plane reductions with input sun gears, fixed ring gears and multiple planet gears. The planet gears are placed symmetrically about the concentric input and output shafts as shown in Figure 1. Each planet of a reduction is connected to the output arm through a single, ball bearing at its center. Since the input sun gear and fixed ring gear mesh with all the planet gears, a single diametral pitch or module is used for all gears in a reduction as is a single face width.



**Figure 1.**  
**Single Plane Planetary Transmission**

No bearings are included on the input or output shaft since the internal loads in the planetary are balanced on these shafts due to the symmetric placement of the planets. Bearings are needed on these shafts but their placement and loading are based on external considerations.

All transmissions carry the same power and have the same outside diameter which provides a radial ring thickness outside the ring gear teeth of 1.5 times the tooth height.

In this comparison, the input speed and torque are fixed as the ratio is varied. For each design, the planetary system life is maximized subject to the above constraints in addition to constraints on the stresses in the gear teeth and on assembly clearances. The parameters which define each design are the number of teeth on the sun gear,  $N_s$ , the face width of the gears,  $f$ , and the diametral pitch of the gears,  $P_d$ .

## KINEMATICS

In a planetary gear train, the planetary gear ratio is the ratio of the speeds of the input and output shafts. To determine this ratio, one first needs to calculate the gear ratio of each gear mesh in terms of the number of teeth on each gear. The gear ratio of the sun gear mesh with the arm fixed is:

$$n_1 = -\frac{N_{pl}}{N_s} \quad (1)$$

and the gear ratio of the ring gear mesh with the arm fixed is:

$$n_2 = \frac{N_r}{N_{pl}} \quad (2)$$

where the overall transmission gear ratio relative to the arm is:

$$n = n_1 \cdot n_2 \quad (3)$$

and the speed of the output arm relative to the fixed ring is:

$$\omega_o = \frac{\omega_s}{(1-n)} \quad (4)$$

So the planetary gear ratio is:

$$n_a = 1 - n \quad (5)$$

And the speed of each planet gear is:

$$\omega_{pl} = \omega_s \left( \frac{n_1 - n}{n_1(1-n)} \right) = \omega_s \left( \frac{1 - n_2}{1 - n} \right) \quad (6)$$

The planet bearing load cycle speed is the speed of the planet with respect to the arm:

$$\omega_b = \omega_{pl} \left( \frac{n}{n - n_1} \right) \quad (7)$$

For each transmission studied, the planetary gear ratio,  $n_a$ , is fixed and the number of teeth on the sun gear is an independent design parameter. Values for the number of teeth on the sun gear, the gear face width and the diametral pitch are found which maximize the service life for a given transmission size. This requires the number of teeth on the ring gear,  $N_r$ , and on each planet gear,  $N_{pl}$ , to be found in terms of  $n_a$  and  $N_s$ .

The number of teeth on the ring gear is related to the number of teeth on the sun by the gear ratio relative to the arm, since the planets become idlers in this inversion.

$$N_r = -n N_s = (n_a - 1) N_s \quad (8)$$

Since the diameter of the ring gear is equal to the diameter of the sun gear plus twice the diameter of the planet gear, the number of teeth on each planet gear can be calculated by:

$$N_{pl} = \frac{N_r - N_s}{2} = \frac{(n_a - 1 - 1) N_s}{2} = \frac{(n_a - 2) N_s}{2} \quad (9)$$

To keep the number of planet teeth positive, the transmission gear ratio,  $n_a$ , must have a value greater than 2. At 2, the planet gears have no size and the planetary reduction ceases to exist.

To prevent interference among gear teeth of adjacent planet gears, sufficient circumferential clearance must be provided. Requiring the distance between the axes of two adjacent planets to be greater than the

outside diameter of the planet gear by twice the tooth addendum will accomplish this:

$$2 \cdot (R_s + R_{pl}) \cdot \sin\left(\frac{\Sigma}{2}\right) > 2 \cdot (R_{pl} + 2 \cdot a) \quad (10)$$

where  $\Sigma$  is the central angle between two adjacent planet center lines and  $a$  is the addendum of the planet gears.

One additional constraint is needed to allow the planets to be positioned symmetrically around the sun gear. The sum of the number of teeth on the sun and on the ring divided by the number of planets must produce an integer.

$$\frac{N_s + N_r}{n_{pl}} = I \quad (11)$$

### TOOTH STRENGTH

The AGMA model for gear tooth bending uses the Lewis form factor and a stress concentration factor to determine the stress in the tooth for a load at the highest point of single tooth contact (AGMA, 1988). The bending stress model is:

$$\sigma = \frac{F_d \cdot P_d \cdot K_f}{f \cdot Y} \quad (12)$$

where  $F_d$  is the tangential dynamic load on the tooth,  $K_f$  is the stress concentration factor, and  $Y$  is the Lewis form factor based on the geometry of the tooth. Since the Lewis form factor is a function of the tooth shape, it is dependent on the number of teeth on the gear, as is the stress concentration factor.

Large localized stresses occur in the fillets of gear teeth due to the change in the cross-section of the tooth. Although the maximum stress is located closer to the root circle than predicted by Lewis' parabola, the distance between the two locations of maximum stress is relatively small and the stress concentration factor accurately compares the maximum stress in the tooth to the Lewis stress (AGMA, 1988). This method of rapid calculation of bending stress for external gear teeth is extended to include the bending stress in the internal gear teeth of the ring gear (Savage, et al, 1995).

In addition to bending stresses, surface contact stresses can contribute to gear teeth failures. The Hertzian pressure model closely predicts these contact pressures:

$$\sigma_H = \left( \frac{F_d}{\pi \cdot f \cdot \cos\phi} \left( \frac{\frac{1}{\rho_1} + \frac{1}{\rho_2}}{\frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2}} \right) \right)^{1/2} \quad (13)$$

where  $\phi$  is the normal pressure angle of the gear mesh,  $\rho_1$  and  $\rho_2$  are the radii of curvature of the pinion and gear tooth surface at the point of

contact,  $v_1$  and  $v_2$  are the Poisson ratios and  $E_1$  and  $E_2$  are the moduli of elasticity of the materials for the two gears.

Contact pressure near the pitch point leads to gear tooth pitting which limits the life of the gear tooth. Gear tip scoring is another type of failure which is affected by the contact pressure at the gear tooth tip. One model for gear tip scoring includes the pressure times velocity factor, where the sliding velocity at the gear tip is tangent to the tooth surfaces.

### SERVICE LIFE

Surface pitting due to fatigue is the basis for the life model for the bearings, gears and transmission. Fatigue due to this mode of failure has no endurance limit, but has a service life described by a straight line on the log stress versus log cycle S-N curve. This life to load relationship can be written for a specific load,  $F$ , at which the ninety-percent reliability life is  $\ell_{10}$  and which is related to the component dynamic capacity,  $C$ , as:

$$\ell_{10} = a \left( \frac{C}{F} \right)^p \quad (14)$$

Here the component dynamic capacity,  $C$ , is defined as the load that produces a life of one-million cycles with a reliability of ninety-percent and  $a$  is the life adjustment factor. The power,  $p$ , is the load-life exponent which is determined experimentally.

Complementing this load-life relationship, is the two-parameter Weibull distribution for the scatter in life. In this distribution, the reliability,  $R$ , is related to the life,  $\ell$ , as:

$$\ln \left( \frac{1}{R} \right) = \ln \left( \frac{1}{0.9} \right) + \left( \frac{\ell}{\ell_{10}} \right)^b \quad (15)$$

A meaningful estimation of service time is the mean time between overhauls. The mean life for a two-parameter Weibull distribution can be expressed in terms of the gamma function,  $\Gamma$ , as:

$$\ell_{av} = \frac{\ell_{10} \cdot 10^6 \cdot \Gamma(1 + \frac{1}{b})}{60 \cdot \omega_o \cdot [\ln(\frac{1}{0.9})]^{\frac{1}{b}}} \quad (16)$$

including the conversion from million cycles to hours, where  $\omega_o$  is the output speed in RPM.

If the repairs are component repairs, rather than full replacements, then the mean life between overhauls is based directly on the mean lives of the individual components. In this case, the transmission repair rate, which is the reciprocal of the mean life, is the sum of the individual component repair rates. Thus, the transmission mean service life is estimated as the reciprocal of the repair rate:

$$\ell_{av,s} = \frac{1}{\sum \frac{1}{\ell_{av,i}}} \quad (17)$$

### PLANETARY DESIGNS

In considering the effects of the gear ratio on the mean transmission life, the input speed and power were held constant. The input speed was 2,000 RPM for all transmissions which carried a power of 51 horsepower with a fixed input torque of 1,600 pound inches. Each transmission had a maximum ring gear outside diameter of 12 inches. The sun gear mesh and the ring gear mesh both had a normal pressure angle of 20 degrees and the same diametral pitch. All gears were made of high strength steel with a surface material strength of 220 ksi. The Hertzian contact pressure was limited to be less than 180 psi and the tooth bending stresses were limited to be less than 40 ksi. These limits include a total load design factor of 1.5 to adjust the nominal stress calculations of Eq. (12) and (13) to code levels. The PV factor was limited to be less than 50 million psi-ft/min and the gear tooth flash temperature was limited to be less than 200° F. The Weibull slope of the sun gear, the three planet gears, and the ring gear was 2.5. The load-life factor of all five gears was 8.93. The planet bearings were 300 series, single-row ball bearings, with a Weibull slope of 1.1, a load-life factor of 3.0 and a life adjustment factor of 6.

Planetary Design Inequality Constraints  
Table 1

Constraint	Value	Unit	Type
bending stress:sun-planet	40,000,000	psi	upper
full load Hertz stress:sun-planet	180,000,000	psi	upper
gear tip Hertz pressure: sun-planet	180,000,000	psi <sub>6</sub>	upper
PV factor of sun-planet teeth	50.000	10 psi-ft/min	upper
flash temp of sun-planet teeth	200.000	deg. F	upper
sun involute interference	0.001	radians	lower
sun face width to diameter	0.750	ratio	upper
bending stress:planet-ring	40,000,000	psi	upper
full load Hertz stress:planet-ring	180,000,000	psi	upper
gear tip Hertz pressure: planet-ring	180,000,000	psi <sub>6</sub>	upper
PV factor of planet-ring teeth	50.000	10 psi-ft/min	upper
flash temp of planet-ring teeth	200.000	deg. F	upper
involute interference:planet-ring	0.001	radians	lower
planet circumference clearance	0.100	in	lower
bearing diameter	0.400	in	lower
diameter of ring gear	12.000	in <sub>3</sub>	upper
volume of transmission	1000.000	in	upper

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 Design Service Lives

Table 2

planet number	ratio	tooth numbers			face width	pitch	life	pitch	life
		$n_a$	$N_s$	$N_{pl}$	$N_r$				
					f	$P_d$	$\ell_{av}$	$P_d'$	$\ell_{av}'$
three	3.0	60	30	120	1.0	11	1040	10.68	1320
	3.5	48	36	120	1.0	11	2430	10.7	3000
	4.0	45	45	135	1.25	12	4720	11.95	4870
	4.5	40	50	140	1.5	13	3880	12.4	5500
	5.0	36	54	144	1.5	13	4940	12.7	5870
	5.5	36	63	162	1.5	15	4300	14.2	6230
	6.0	30	60	150	1.5	14	3600	13.2	5590
	6.5	24	54	132	1.5	12	3740	11.68	4560
	7.0	24	60	144	1.5	13	3870	12.7	4600
	7.5	24	66	156	1.5	14	3900	13.7	4580
four	8.0	24	72	168	1.5	15	3810	14.7	4450
	3.0	60	30	120	1.0	11	1850	10.68	2340
	3.5	40	30	100	1.0	10	1640	9.02	3680
	4.0	40	40	120	1.25	11	5880	10.7	7240
	4.5	40	50	140	1.5	13	6900	12.35	10100
five	5.0	36	54	144	1.5	13	8780	12.68	10560
	3.0	60	30	120	1.0	11	2890	10.68	3660
	3.5	40	30	100	1.0	10	2570	9.05	5600
	4.0	40	40	120	1.25	11	9180	11.7	11300

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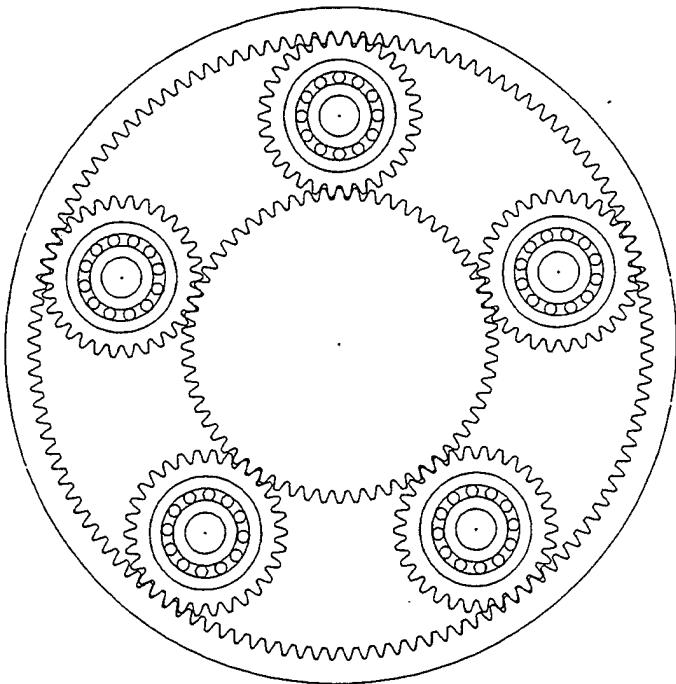
Table 2 lists the obtained designs with the numbers of teeth on the sun, planet and ring gears, the gear face width and the diametral pitch for each ratio. These teeth numbers are discrete values which produce the required planetary ratio and allow symmetric placement of the planets for radial load cancellation. After the diametral pitch, the mean service life of the transmission is listed for component replacement at repair. This life corresponds to the integer diametral pitch listed before it. It also corresponds to a somewhat smaller transmission as dictated by the integer pitch. The last two columns show larger lives which vary more continuously and the fractional diametral pitch required to obtain these lives by allowing the transmission to have the full 12 inch outside ring diameter. The table includes blocks of data for three, four and five planet designs.

Even higher lives would be possible with fine pitch gearing since the outside diameter limit includes the ring gear dedendum and the rim height outside the ring gear pitch diameter. Both distances are proportional to the tooth height. However, the diametral pitch is limited to be 16 or less to maintain overload tooth bending strength.

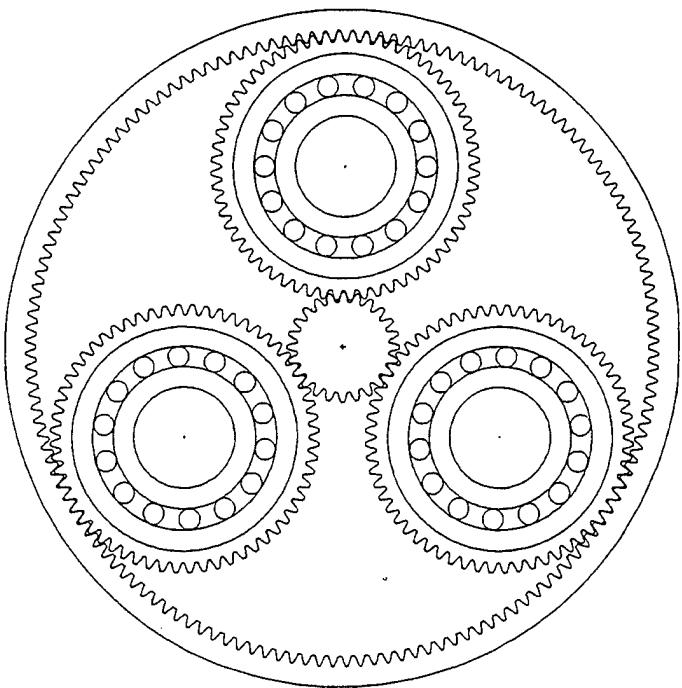
The results show that as the gear ratio was increased, the size of the sun gear decreased and the size of the planet gears increased. Figures 2, 3, and 4 show planetary transmission designs for speed reduction ratios of three, five and seven respectively.

The effect of the gear ratio on the mean life of the transmission is plotted in Figure 5. For the integer diametral pitch designs with three planets, the mean service life, plotted as a series of crosses, increased from 1040 hours for a gear ratio of three to 4940 hours for a gear ratio of five, and then decreases to 3600 hours for a gear ratio of six, with a final life of 3810 hours for a gear ratio of eight. Higher lives which varied more continuously were available with uneven pitches and are plotted as a life limit line above the found design lives. This line corresponds to the primed pitches and lives of table 2 and is also jagged due to the discrete nature of the numbers of teeth.

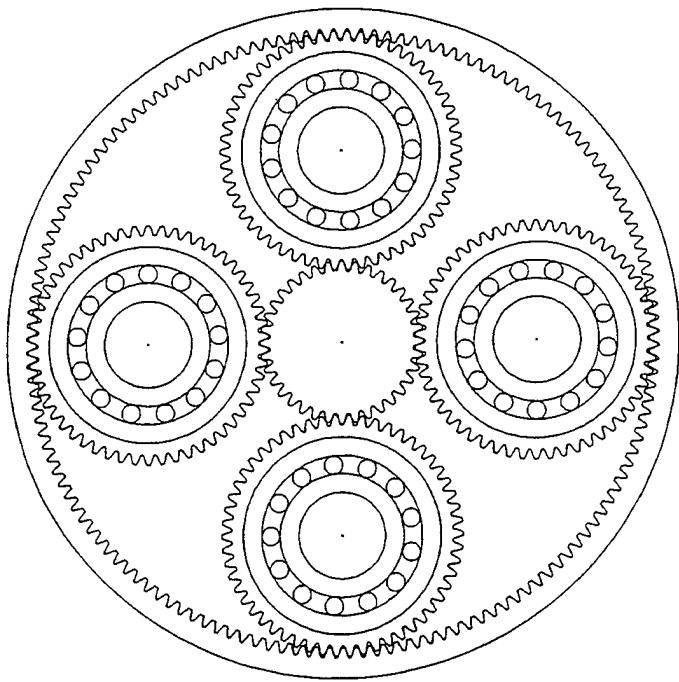
Similar data is plotted with circles for integer pitch designs with four planets and with squares for designs with five planets. For the four planet designs, the integer pitch design lives ranged from 1850 hours for a gear ratio of three to a maximum of 8780 hours for a gear ratio of five. And for the five planet designs, the mean service lives varied from 2890 hours for a gear ratio of three to 9180 hours for a gear ratio of four. Similar life limit designs are plotted above these points for designs with the full twelve inch outside diameter and non-integer diametral pitches.



**Figure 2.**  
Planetary Transmission with a Reduction Ration of 3.0

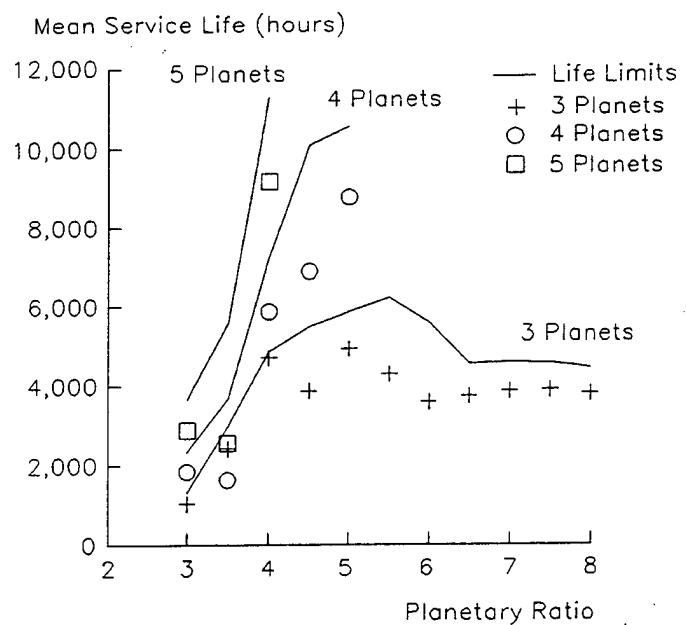


**Figure 4.**  
Planetary Transmission with a Reduction Ration of 7.0



**Figure 3.**  
Planetary Transmission with a Reduction Ration of 5.0

At low planetary ratios, the planet and planet bearing sizes were small. At a ratio of three, the smallest bearings for the optimal designs were selected, causing the low life designs for each number of planets.



**Figure 5.**  
Mean Transmission Service Life Versus Speed Reduction Ratio with Constant Input Speed and Torque

As the planetary ratio was increased, the size of the planets and the planet bearings increased, which increased the life of the transmissions. With more planets to share the load, the four and five planet designs had

greater lives than the three planet designs. However, circumferential planet interference limited the five planet designs to a maximum ratio of four and the four planet designs to a maximum ratio of five. At ratios above five and a half, the life of the three planet designs dropped due to the increase in the output torque. Once again, the lower transmission life was attributable to lower planet bearing life. At a gear ratio of eight, the pitch diameter of the sun gear had decreased to 1.6 inches with a face width of 1.5 inches. Larger ratios would have decreased this length to diameter ratio even further and would have increased the bending stress in the sun gear teeth above the 21 ksi present in the eight to one gear ratio design. So the table and graph were cut off at this gear ratio even though designs are possible at higher ratios with three planets.

## CONCLUSIONS

The effect of the gear ratio on the life of the transmission was examined. Of interest is the possibility of an optimal planetary gear reduction from a life standpoint. In this study the overall size of the transmission was held constant, its strengths were maintained and the ratio was varied for the three, four and five planet arrangements. Each optimal design was defined by the number of teeth on the sun gear, the gear face width and the diametral pitch of the gears. For the comparison, the transmission input speed and power was held constant. The results show that as the gear ratio increased, the size of the sun gear decreased and the size of the planet gears increased. At a ratio of three, the planet bearings were reduced in size relative to the transmission sufficiently to limit the transmission life. Five planet designs had a maximum ratio of four with no planet interference, and four planet designs could be obtained with ratios up to five. Above five and a half, the lives of the three planet designs fell off due to the higher output torques. The optimal design exists for a transmission with a gear ratio of approximately four to five.

## REFERENCES

- AGMA STANDARD, 1988, "Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth," ANSI/AGMA 2001-B88, Alexandria, Virginia.
- Chin, H., Danai, K., and Lewicki, D. G., 1995, "Fault Detection of Helicopter Gearboxes Using the Multi-Valued Influence Matrix Method," ASME Journal of Mechanical Design, Vol. 117, No. 2, pp. 248-253.
- Donley, Mark G. and Steyer, Glen C., 1992, "Dynamic Analysis of a Planetary Gear System," Advancing Power Transmission into the 21st Century, ASME DE-VOL-43-1, pp. 117-127.

Fox, Paul F., 1991, "Gear Arrangements," Ch. 3, Dudley's Gear Handbook, 2nd Ed., D. P. Townsend, Ed. McGraw Hill, New York.

Kahraman, A., 1994, "Planetary Gear Train Dynamics," ASME Journal of Mechanical Design, Vol. 116, No. 3, pp. 713-720.

Lynwander, Peter, 1983, Gear Drive Systems, Marcel Dekker, New York.

Muller, Herbert W., 1982, Epicyclic Drive Trains, Wayne State University Press, Detroit, Mich.

Rashidi, Majid and Krantz, Timothy L., 1992, "Dynamics of a Split Torque Helicopter Transmission," Advancing Power Transmission into the 21st Century, ASME DE-VOL-43-1, pp. 19-33.

Rubadeux, K. L., 1995, "PLANOPT - A Fortran Optimization Program for Planetary Transmission Design," M.S. Thesis, The University of Akron, Akron, Ohio.

Saada, A. and Velex, P., 1995, "An Extended Model for the Analysis of the Dynamic Behavior of Planetary Trains," ASME Journal of Mechanical Design, Vol. 117, No. 2, pp. 241-247.

Savage, M., Radil, K. C., Lewicki D. G., & Coy, J. J., 1989, "Computerized Life and Reliability Modeling for Turboprop Transmissions," AIAA Journal of Propulsion and Power, Vol. 5, No. 5, pp. 610-614.

Savage, M. and Lewicki, D. G., 1991, "Transmission Overhaul and Component Replacement Predictions Using Weibull and Renewal Theory," AIAA Journal of Propulsion and Power, Vol. 7, No. 6, pp. 1049-1054.

Savage, M., Mackulin, B. J., Coe, H. H., and Coy, J. J., 1992, "Maximum Life Spur Gear Design," AIAA Journal of Propulsion and Power, Vol. 8, No. 6, pp. 1273-1281.

Savage, M., Prasanna, M. G., and Coe, H. H., 1993, "Maximum Life Spiral Bevel Reduction Design," Gear Technology, The Journal of Gear Manufacturing, Vol. 10, No. 5, pp. 24-40.

Savage, M., Rubadeux, K. L., Coe, H. H., and Coy, J. J., 1994, "Spur, Helical and Spiral Bevel Transmission Life Modeling," NASA TM 106552, AIAA 94-3079, 30th Joint Propulsion Conference, Indianapolis, Indiana.

Savage, M., Lattime, S. B., Kimmel J. A., and Coe, H. H., 1994, "Optimal Design of Compact Spur Gear Reductions," ASME Journal of Mechanical Design, Vol. 116, No. 3, pp. 690-696.

Savage, M., Rubadeux, K. L., and Coe, H. H., 1995, "Bending Strength Model for Internal Spur Gear Teeth," NASA TM 107012, AIAA 95-3049, 31st Joint Propulsion Conference, San Diego, California.

Tsai, L. W., 1987, "An Application of the Linkage Characteristic Polynomial to the Topographical Synthesis of Epicyclic Gear Trains," ASME Journal of Mechanisms, Transmissions and Automation in Design, Vol. 109, No. 3, pp. 329-337.

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<b>13. ABSTRACT (Maximum 200 words)</b>  Planetary gear transmissions are compact, high-power speed reductions which use parallel load paths. The range of possible reduction ratios is bounded from below and above by limits on the relative size of the planet gears. For a single plane transmission, the planet gear has no size at a ratio of two. As the ratio increases, so does the size of the planets relative to the sizes of the sun and ring. Which ratio is best for a planetary reduction can be resolved by studying a series of optimal designs. In this series, each design is obtained by maximizing the service life for a planetary with a fixed size, gear ratio, input speed, power and materials. The planetary gear reduction service life is modeled as a function of the two-parameter Weibull distributed service lives of the bearings and gears in the reduction. Planet bearing life strongly influences the optimal reduction lives which point to an optimal planetary reduction ratio in the neighborhood of four to five.						
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